Exercise Report
(Sound and Vibration)

Exercise B.
Measurement of Power Input, Vibration and Damping of a Box Structure

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1. Introduction

The purpose of this laboratory exercise is to measure and analyze different quantities (e.g. mean-value responses) in a complex vibro-acoustic structure.

The problem one has to face when dealing with complicated structures is that the exact analysis of such quantities implies a large computational load due to the fact that the model has to be developed and the boundary conditions set.

Instead, it has been shown that the simplified method named Statistical Energy Analysis (SEA) is powerful enough for predicting the responses of a complex system. Hence, in the present exercise we will make use of this method to analyze, among other magnitudes, the vibratory input power, the mean-square velocity and the damping of a system.
2. Theory

The following theory is stated according to [Ohlrich, 2005] and [Ohlrich, 2006].

2.1. Input Power

Injection of power can lead to vibration of structures. Therefore, the input power of a system is often an important reference when vibration analysis on complicated structures is carried out. By definition, the instantaneous power is defined as the product between force and velocity. As these two magnitudes are typically not constant in time, rather fluctuating, it is usually interesting to determine the time-average power instead:

\[ P_s = P_s(t) = F_0(t)v_0(t) = \ldots = \frac{1}{T} \int_{0}^{T} F_0(t)v_0(t) \, dt = \Re \{ Y_{00} \} , \]

where \( Y_{00} \) is the input mobility of the system. Therefore, by knowing the mean-square value of the force and the input mobility of the system it is possible to calculate the average power. Note that both magnitudes are frequency dependent. In fact, this product can be determined for each frequency component of concern by means of the time-average autospectrum of the force, \( G_{FF}(f) \):

\[ P_s(f) = G_{FF}(f) \Re \{ Y_{00}(f) \} , \]

and thus, the following equation leads to the total average input power injected to the system:

\[ P_s = \int_{0}^{\Delta f} G_{FF}(f) \Re \{ Y_{00}(f) \} \, df. \]

2.2. System Loss Factor

Another parameter that describes the performance of a system is the damping loss factor, \( \eta \). It is possible to determine a mathematical expression for it based on energy and power considerations. The energy balance equation states that the difference between the input and the dissipated power of the system is equal to the variation of the energy in the system:

\[ P_s(t) - P_{\text{diss}}(t) = \frac{dE(t)}{dt} . \]

This equation shows that the input power and the dissipated power are equal once the steady state is reached. Generally, the loss factor is defined as:

\[ \eta = \frac{1}{2\pi} \frac{\Delta E_{\text{diss}}}{E} , \]

where \( \Delta E_{\text{diss}} \) and \( E \) are the dissipated and the total stored energy of the system in a single cycle (\( T \)). For harmonic excitation at a certain angular frequency \( \omega \), the latter equation can be extended taking into account the relation between power and energy as well as the conclusion deduced from equation (4) in steady state:

\[ \eta = \frac{1}{2\pi} \frac{\Delta E_{\text{diss}}}{E} = \frac{1}{2\pi} \frac{P_{\text{diss}} T}{E} = \frac{1}{2\pi} \frac{P_s}{E} \frac{2\pi}{\omega} = \frac{P_s}{\omega E} . \]
In addition, the time-average reversible energy stored in the system is the summation of the time-average of kinetic and potential energy:

\[ \bar{E} = \bar{E}_{\text{kin}} + \bar{E}_{\text{pot}} = \bar{E}_{\text{kin}} + 2\bar{E}_{\text{pot}} = 2\bar{E}_{\text{kin}}. \] (7)

The latter result can be used to extend the expression of the loss factor further:

\[ \eta = \frac{P_s}{\omega \bar{E}} = \frac{P_s}{2\omega \bar{E}_{\text{kin}}} = \frac{P_s}{\omega M \bar{v}^2}, \] (8)

where \( P_s \) and \( \bar{v}^2 \) are the input power and the mean-square value of the velocity of the system for a certain angular frequency \( \omega \), and on the other hand, \( M \) is the total mass of the system.

Alternatively, the loss factor at a certain resonance frequency, \( f_0 \), can be calculated as follows:

\[ \eta = \frac{f_u - f_l}{f_0}, \] (9)

where \( f_u \) and \( f_l \) are the upper and lower cutoff frequencies of the resonance. These frequencies can be determined from either the half-power-point bandwidth (3 dB-bandwidth) or the Nyquist diagram. The latter is particularly suitable in order to examine the resolution of the measurement as well as the accuracy of the estimated loss factor.

2.3. Temporal and Spatial Average Considerations

Temporally and spatially averaged mean-square values are often of interest when dealing with complicated structures. For instance, the temporally and spatially averaged mean-square velocity is defined as:

\[ \langle \bar{v}^2 \rangle = \frac{1}{S} \int_S \bar{v}^2(x, z, t) \, dS \approx \frac{1}{m} \sum_{j=1}^{m} \bar{v}^2_j, \] (10)

where the approximation is the practical implementation for a substructure with constant thickness and where \( m \) is the number of discrete measuring points that are chosen randomly across the surface of concern. In addition, the total mean-square velocity of the whole structure composed of several substructures can be calculated as follows:

\[ \langle \bar{v}^2 \rangle = \frac{1}{M} \sum_{i=1}^{n} M_i \cdot \langle \bar{v}^2_i \rangle, \] (11)

where \( M_i \) and \( \langle \bar{v}^2_i \rangle \) are the mass and the mean-square velocity of the \( i \)'th substructure and \( M \) is the total mass.

Likewise, the average transfer mobility of a structure composed of \( n \) substructures and excited by a point-force source is:

\[ \left( \frac{\langle |Y| \rangle^2}{F_0^2} \right)^{1-n} = \frac{\sum_{i=1}^{n} M_i \cdot \langle \bar{v}^2_i \rangle}{F_0^2 \cdot \sum_{i=1}^{n} M_i} \] (12)
Finally, the loss factor of the complete structure can be calculated from equation (8) when temporally and spatially averaged mean-square values of the supplied power and the velocities of each substructure are measured:

$$\eta(\omega) = \frac{P_s(\omega)}{\omega \sum_{i=1}^{n} M_i \cdot \langle v_i^2 \rangle}.$$  

(13)

### 2.4. Coupled Systems

Complicated structures are often difficult to analyze: Boundary conditions are not sufficiently well-known, the input of system is often broadband, so many natural modes are excited at the same time, etc. This leads to complicated solutions that involve a high computational load. Alternatively, a strongly simplified method called Statistical Energy Analysis (SEA) has been developed. It turned out that this is an accurate method for predicting mean value responses as long as the modal density is high enough to ensure the use of statistics.

From this point of view, complicated systems can be split into simplified subsystems that are coupled to each other. It is possible to define the interaction between these subsystems by means of power balance equations. In this sense, the distribution of vibratory energy of two coupled substructures can be described by the ratio of their mean-square velocities. For strongly coupled systems, i.e. when there is less power dissipated in the secondary system than there is power returned to the original one ($\eta_2 \ll \eta_1$), this ratio can be calculated as follows:

$$\frac{\langle v_2^2 \rangle}{\langle v_1^2 \rangle} = \frac{\text{Re} \{ Y_2 \}}{\text{Re} \{ Y_1 \}} = \frac{M_1 \cdot \Delta N_2}{M_2 \cdot \Delta N_1},$$

(14)

where $\Delta N_1$ and $\Delta N_2$ are the numbers of modes in the frequency band of concern.

This equation can be simplified for substructures that consist of plane and homogeneous plates of the same material:

$$\frac{\langle v_2^2 \rangle}{\langle v_1^2 \rangle} = \ldots = \left( \frac{h_1}{h_2} \right)^2,$$

(15)

In addition, provided that the dimensions of the structure are larger than the considered wavelength, the time-average power supplied by a vibration source can be regarded to be independent of the kind and shape of the structure. This means that a coarse approximation of the supplied power can be calculated as follows:

$$P_s = F_0^2(t) \text{ Re} \{ Y_{00} \} \approx F_0^2(t) \text{ Re} \{ Y_{\infty} \},$$

(16)

i.e. the injected power is the same as the power that is injected to an infinite plate. This leads to the following approximation:

$$\text{Re} \{ Y_{00} \} \approx \text{Re} \{ Y_{\infty} \} = \frac{1}{8\sqrt{B'\rho h}},$$

(17)

This is an analogy based on statistical room acoustics: The acoustical power $P_a$ radiated by a source into a room can be regarded to be the same as it would radiate under free-field conditions, if the volume of the room is big enough compared to the wavelength and the losses of the room are moderate.
where $B'$ is the plate bending stiffness per unit width, $\rho$ is the mass density of the plate and $h$ is its thickness.
3. Measurement Procedure

3.1. Device List

**Brüel & Kjær PULSE Analyzer System Type 3560-B-130.** Data acquisition system with 5 inputs and one generator output.

**Brüel & Kjær NEXUS Conditioning Amplifier Type 2692.** Four-channel conditioning amplifier.

**Brüel & Kjær Calibrator Type 4294.** Portable electrodynamic vibration calibrator that produces a tone at 159.2 Hz with a RMS velocity of 10 mm/s, i.e. a velocity level of 140 dB re $10^{-9}$ m/s.

**Brüel & Kjær Force Transducer Type 8200.** Force transducer with a reference sensitivity of 3.85 pC/N. Its weight is 21 g.

**Brüel & Kjær Accelerometer Type 4393.** Piezoelectric charge accelerometer that weighs 2.4 g. Its charge sensitivity (@ 159.2 Hz) is $3.1 \pm 2\%$ pC/g.

**Brüel & Kjær Type Mini-Shaker 4810.** Shaker used for vibration testing of small objects. Its maximum force is 7 or 10 N depending on the frequency which can be up to 18 kHz.

**Amplifier.** Exciter preamplifier.

**Computer.** To process the data from the PULSE. The used software is MATLAB.

These devices can be seen in figure 1 where the setup of the experiment is sketched. The system is driven with a shaker which is controlled with the analyzer. The measured signals from the force transducer and the accelerometer are amplified with the NEXUS and sent to the analyzer. Different parameters, such as the cross-spectrum of the force and velocity or the input mobility are measured, stored and afterwards processed with MATLAB.

![Figure 1: Experiment setup.](image-url)
3.2. Calibration

Before starting the measurements, the calibration of the force transducer and the accelerometer must be performed. However, due to time limitations, only the calibration of the accelerometer is carried out.

This is done by connecting the accelerometer to an electrodynamic vibration calibrator, and setting the amplifiers and the analyzer as follows:

- The sensitivities of the force transducer and the accelerometer are adjusted for the corresponding channels in the NEXUS amplifier. Furthermore, an amplification of the output signal is applied in order to insure a good signal-to-noise ratio in the analyzer software. In case of the velocity signal this is done when measuring the input mobility, i.e. at the position where the largest vibration amplitudes are given.
- The analyzer has to be set to ‘Dual Channel’, ‘Base Band’ 800 Hz, ‘Flat Top’ filter only for calibration.
- The amplifier for the excitation signal is set to ‘Cal.’.

It was verified that the RMS output of the integrated accelerometer signal, when attached to the calibrator, corresponds to a level of 140 dB re $10^{-9}$ m/s.

3.3. Procedure
3.3.1. Measurements

A sketch of the box-shaped structure under test is shown in figure 2. It consists of a frame formed by the surfaces $S_1$ to $S_4$, which are made of 3 mm thick steel and are welded together. The fifth panel (having the area $S_5$) is made of unknown material and has a thickness of 1.5 mm. It is screwed to the previously described frame along all four edges. The areas of the box is summarized in table 1.

The exciter (shaker) is situated inside the box, namely on panel 1. It is coupled to the latter via a force transducer, i.e. the inserted force is measured directly at the excitation point. White noise is used as an excitation signal. As described in section 2.1, the force auto-spectrum will be needed in order to compute the input...


<table>
<thead>
<tr>
<th>Panel #</th>
<th>Area in m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 and 2</td>
<td>( S_1 = S_2 = 0.109 )</td>
</tr>
<tr>
<td>3 and 4</td>
<td>( S_3 = S_4 = 0.176 )</td>
</tr>
<tr>
<td>5</td>
<td>( S_5 = 0.246 )</td>
</tr>
</tbody>
</table>

The velocity response of each of the panels is measured by means of a small accelerometer (2.4 g in order to minimize the additional mass load on each of the panels). In order to measure the mobility, the accelerometer signal is integrated, which yields the velocity instead of the acceleration. As the mentioned white noise is an aperiodic signal, the Hanning window is used when estimating the transfer function between velocity and force (i.e., the mobility). A frequency range from 20 to 6.4 kHz is chosen, using a frequency resolution of 1 Hz. However, when the properties of a single mode are studied, these settings are changed in order to achieve a higher frequency resolution, i.e., \(< 1 \) Hz. Each recording is an averaging process over several window lengths. A measurement time around 20 s per transfer function ensures the sufficient reduction of uncorrelated components, e.g., uncorrelated excitation by airborne background noise.

Besides this temporal averaging, a spatial averaging per panel is also carried out. For this purpose, the measurement is interrupted after the temporal average for one accelerometer position is determined. Then it is placed at a different location on the panel and the measurement continues. In this way, an average over seven positions per panel is recorded. However, in the case of panel 1 (i.e., the panel on which the excitation takes place) care has to be taken not to measure the velocity too close to the excitation point. Otherwise, only the input mobility would be captured at this point.

Because the force is measured as \( G_{FF}(f) \), equation (3) has to be modified before the input power can be calculated. Equation (3) can be transferred into discrete signals to

\[
P_S = \sum G_{FF}(f) \cdot \text{Re}\left\{Y_{00}(f)\right\} \cdot \text{resolution}
\]

where the resolution is 1 Hz in this exercise.

The corresponding source code for these calculations can be found in appendix B.1.

### 3.3.2. ‘Theoretical’ Calculations

After processing all the measured data, some ‘theoretical’ calculations are carried out. On the one hand, a coarse frequency-band averaged estimate for the modal frequency range of the real part of the input mobility can be calculated from (17). Because (17) is only valid, when the wavelength is smaller than the dimensions of the structure. Thus, the frequency range of interest is \( f > 100 \) Hz.

On the other hand, the average transfer mobility of subsystem 1 (panel 1-4) can be calculated by using SEA for resonant subsystem vibrations. The approximate
expression is
\[ \langle |Y_t|^2 \rangle_{1-4} = \frac{\text{Re} \{ Y_{00} \}}{\omega M \eta_{\text{tot}} \left( 1 + \frac{\eta_2}{\eta_{\text{tot}}} \cdot \frac{S_2 \cdot h_2 \cdot c_{11,2}}{\sum_{i=1}^{4} S_i \cdot h_i \cdot c_{i1,1}} \right)}, \] (19)

where:
- \( M \) is the total mass of subsystem 1,
- \( \eta_{\text{tot}} \) is the total damping loss factor of the whole system,
- \( \eta_2 \) is the total damping loss factor for subsystem 2,
- \( h_1 \) and \( h_2 \) are the thicknesses of the plates of subsystems 1 and 2 respectively,
- \( c_{11,1} \) and \( c_{11,2} \) are the speeds of sound in the plates of subsystems 1 and 2,
- \( S_i \) is the surface of the \( i \)th plate of subsystem 1,
- and \( S_2 \) is the surface of the plate of subsystem 2.

In the present exercise the used values are: \( h_1 = 3 \) mm, \( h_2 = 1.5 \) mm, \( \eta_1 = 0.01 \) and \( \eta_2 = 0.003 \), and the calculations have been implemented in MATLAB. For clarity’s sake, the source code is attached in appendix B.2 on page 25.
4. Results and Discussion

4.1. Input Power

The measured force auto-spectrum $G_{FF}(f)$ can be seen in figure 3. It can be seen that it only slightly resembles the auto-spectrum for white noise which is flat for all frequencies. This is because the structure represents a load on the shaker. This means that depending on the frequency it can become very easy or very hard to drive the system. Therefore, the actually transmitted force varies with frequency. In this case, the load generally increases at high frequencies which makes the force decrease more and more.

The magnitude and phase of the measured input mobility can be seen on figure 4 and 5 respectively.

From figure 4 it can be seen that at frequencies below 32 Hz the measured input mobility is mass-like, and from 32 Hz to around 100 Hz the input mobility is spring-like. The change from mass-like to spring-like can also be seen in figure 5 where there is a $\pi$ phase shift from $-\pi/2$ to $\pi/2$ at that frequency.

Equation (2) has been implemented in Matlab and the input power can be seen in figure 6.

Using equation (18) the total power is calculated to be

$$P_S = 2.1 \text{ mW}. \quad (20)$$
Figure 4: Magnitude of measured input mobility $Y_{00}(f)$.

Figure 5: Phase of measured input mobility $Y_{00}(f)$. 
Figure 6: Input power as a function of frequency.
4.2. Mean-square Velocity of Panels

The auto-spectrum of the spatially average mean-square velocity $\langle G_{vv}(f) \rangle$ was measured by averaging seven positions for each panel. Measurement results for subsystem 1 (panel 1-4) and subsystem 2 can be seen in figure 7 and figure 8 respectively. The magnitude is decreasing at high frequencies because, as it has been shown in figure 3, the magnitude of the force auto-spectrum also decreases at high frequencies. Further into details in figure 7 it can be seen that panels 3 and 4 present very similar responses since they have the same dimensions and occupy symmetric positions in the system. Panels 1 and 2 are also symmetrical, but the panel 1 is excited directly by the force point source. This leads to a more compressed dynamic range for panel 1 compared to the other panels due to the fact that part of the force

![Figure 7](image1.png)

**Figure 7:** Measured auto-spectrum $\langle G_{vv}(f) \rangle$ for subsystem 1.

![Figure 8](image2.png)

**Figure 8:** Measured auto-spectrum $\langle G_{vv}(f) \rangle$ for subsystem 2.
vanishes before reaching the panels 2–4.

As can be seen in figure 8, the magnitude of the velocity auto-spectrum for plate 5 is higher than the plates measured from subsystem 1. This is because the subsystems are strongly coupled and the plate is thinner for subsystem 2, which means that the plate can move more freely, hence it presents higher velocity.

4.3. **Average Transfer Mobility**

Subsystem 1 can be regarded as a system composed of four substructures. Therefore, the average transfer mobility of subsystem 1 is calculated according to equation (12). This is shown in figure 9. Graphically, this figure is just and average of all the panels shown in figure 7, \( \langle \overline{v^2} \rangle \), divided by the force auto-spectrum, \( G_{FF}(f) \).

![Average transfer mobility for panels 1 to 4.](image)

**Figure 9:** Average transfer mobility for panels 1 to 4.

4.4. **Mass Density Determination for Subsystem 2**

As has been mentioned in section 2.4, the unknown density for plate 5 \( \rho_5 \) can be compared to the density for subsystem 1 by assuming that the subsystems are ‘strongly coupled’. Specifically, the approximation stated in equation (15) determines whether the material (density) is the same for both subsystems. For the present exercise, subsystem 1 consists of four plates with thickness of \( 3 \text{ mm} \) and subsystem 2 consists of one plate with thickness of \( 1.5 \text{ mm} \). Thereby (15) equals

\[
\left( \frac{h_1}{h_2} \right)^2 = \left( \frac{3 \cdot 10^{-3}}{1.5 \cdot 10^{-3}} \right)^2 = 4 \quad \Rightarrow \quad 10 \cdot \log_{10} \left( \frac{4}{1} \right) = 6.021 \text{ dB.} \tag{21}
\]

The average mean-square velocity for subsystem 1 and subsystem 2 has been frequency-band averaged over 1/3 octave bands and divided. The results as a func-
tion of frequency can be seen in figure 10. From this figure, it can be seen that the mean-square velocity ratio is close to 6 dB but differs due sources of error from the fact that practice only resembles theory. In fact, the assumption of strongly coupled systems seems to be not totally fulfilled. However, because of the resembles of \( \langle \bar{v}_2^2 \rangle / \langle \bar{v}_1^2 \rangle \) and \( h_1^2 / h_2^2 \) and by looking at the plates, the material has been assessed to be steel with the same density for both subsystems.

4.5. System Loss Factor

The loss factor has been obtained according to (13). The corresponding result is shown as a function of frequency in figure 11, where the average value is given on top of the figure. Although there is a peak around 200 Hz, the loss factor remains approximately constant for the rest of frequencies. In fact, the average loss factor is 0.01 when averaging over the frequency range from 100 – 6400 Hz. As mentioned previously, the frequency range of concern is above 100 Hz since the theory used for such a calculation relies on a high modal density.

In addition, the damping loss factor for a certain mode can be calculated from the measured input mobility by means of either the 3 dB bandwidth method or the Nyquist diagram. This has been done for two different modes centered at 100.60 Hz and 911.94 Hz. Table 2 states the results for such analysis. The Nyquist diagram for a mode near 900 Hz can be seen in figure 12. The fact that part (b) reveals a circle indicates both the presence of a single mode and a sufficient frequency resolution.

Note that the loss factor measured for modes 100.60 Hz and 911.94 Hz corresponds approximately to the values shown for these frequencies in figure 11.

\[ \text{See figure 15 in appendix A for the results without being averaged over third octave bands.} \]
Figure 11: Measured loss factor.

Figure 12: The mode near 900 Hz highlighted in figure 12a is plotted in the Nyquist diagram in figure 12b.
Table 2: Damping loss factors estimated from the measured input mobility.

<table>
<thead>
<tr>
<th>Frequency in Hz</th>
<th>Loss factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 )</td>
<td>( f_u )</td>
</tr>
<tr>
<td>100.60</td>
<td>100.90</td>
</tr>
<tr>
<td>911.94</td>
<td>917.94</td>
</tr>
</tbody>
</table>

4.6. Comparison Between Experimental and ‘Theoretical’ Results

A coarse frequency-band averaged estimate for the modal frequency range of the real part of the input mobility can be calculated from equation (17).

In figure 13, a comparison between the real part of the measured input mobility and the coarse estimate can be seen, with and without applying third octave band averages. As it can be seen, both measurement curves differ from the coarse estimate.

The reason is that this coarse approximation assumes that the structure can be regarded as an infinite plate. However, on average the values are comparable.

Likewise, in figure 14, a comparison between the ‘theoretical’ and measured average transfer mobility of subsystem 1 can be seen for \( f > 100 \) Hz. In this case, the measured results fit the approximation better. The ‘theoretical’ slope is acceptably represented.
Figure 14: Measured and ‘theoretical’ average transfer mobility.
5. Conclusions

In the present report, SEA approximations have been investigated by comparing calculations and measurements from a complex system consisting of two subsystems. The presented results yield the following conclusions:

- **Mechanical properties can be determined from vibration measurements.** It is, for example, possible to determine the damping loss factor for one mode from a Nyquist plot, and calculate the damping loss factor for all frequencies from energy and power considerations.

- **Theoretical calculations based on SEA are in agreement with the measurements to a certain degree.** Therefore, this method can be applied to predict the responses of a complex vibrational system.

- **When using SEA the approximations should be taken into consideration before relying on the predictions.**

- **The method described in this report can be used to check whether two subsystems, which consist of plane and homogeneous plates, are made of the same material.**

We herewith confirm the unassisted creation of this exercise report. All authors contributed in equal amount.

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A. Further Information

Figure 15: Average mean-square velocity for $\langle v_2^2 \rangle / \langle v_1^2 \rangle$. 

\[
\frac{\langle v_2^2 \rangle}{\langle v_1^2 \rangle} (\text{dB}) 
\]

\[
\begin{array}{cccccc}
125 & 250 & 500 & 1k & 2k & 4k \\
-25 & -20 & -15 & -10 & -5 & 0
\end{array}
\]
B. Source Code

B.1. measurements_script.m

```matlab
1: clear;
2: close all;
3: clc;
4: % Basic variables:
5: rho1 = 7.8e3; %kg/m^3, valid for plate 1 to 4
6: rho2 = 7.8e3; %kg/m^3, valid for plate 5
7: rho = [rho1 rho1 rho1 rho1 rho2];
8: S = [0.109 0.109 0.176 0.176 0.246]; % m^2
9: h = [3e-3 3e-3 3e-3 3e-3 1.5e-3]; % m
10:
11:
12: % Autospectrum of velocity Gvv
13: file_vv = {'../Measurements/last_trial/Autospectrum(Velocit...'};
14: titles = {'Velocity plate 1','Velocity plate 2','Velocity plate 3',...
15: 'Velocity plate 4','Velocity plate 5'};
16:
17: for ii = 1:length(file_vv)
18: figure;
19: [bla,f,Gvv(ii,:)] = read_pulse_mod(char(file_vv(ii)));
20: semilogx(f,10*log10(Gvv(ii,:)/(10^-9)^2),'LineWidth',1.0, 'color',[0.6
21: 0.6 0.6]);
22: %title(char(titles(ii)));
23: nicefigure;
24: xlim([20 6400]);
25: ylim([60 120]);
26: nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
27: xlabel('Frequency (hertz)');
28: ylabel('$G_{vv}(f)$ (deci\bel\re \unit{10^{-9}}{\meter/\second})');
29: printlatex(sprintf('../Report/graphics/Mean_square_velocity_Panel_%d',
30: ii),6.5,8,'nofigcopy');
31: end
32:
33: % Autospectrum of Force Gff
34: file_ff = {'../Measurements/last_trial/Autospectrum(Force transducer) -
35: InputRepetition.txt'};
36:
37: for ii=1:length(file_ff)
38: figure;
39: [bla,f,Gff(ii,:)] = read_pulse_mod(char(file_ff(ii)));
40: semilogx(f,10*log10(Gff(ii,:)/(10^-6)^2),'linewidth',1.2,'color',[0.6
41: 0.6 0.6]);
42: nicefigure;
43: xlim([20 6400]);
44: nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
45: xlabel('Frequency (hertz)');
```
% Input mobility
file_H1 = {'../Measurements/Frequency Response H1(Velocity (acc.),Force transducer) - Zoom.txt',
            '../Measurements/Frequency Response H1(Velocity (acc.),Force transducer) - Input.txt'};

for ii=1:length(file_H1)
    figure;
    [bla,f,real_H1(ii,:),img_H1(ii,:)] = read_pulse(char(file_H1(ii)));
    H1(ii,:) = real_H1(ii,:) +i*img_H1(ii,:);
    if ii==1
        hh=area([881 937], [-80 40; -80 40]);
        set(hh(2),'FaceColor',[0.85 0.85 0.85],'LineStyle','--','LineWidth',1.2);
        hold on;
        semilogx(f,20*log10(abs(H1(ii,:))),'k','linewidth',1);
        xlabel('Frequency (hertz)');
        ylabel('$|\mathbf{Y}_{00}(f)|^2$ (deci\,bel)');
        xlim([700 f(end)]);
        ylim([-80 -40]);
        nicelogx([700 800 900 1000 2000],'k');
    else
        semilogx(f,20*log10(abs(H1(ii,:))),'linewidth',1.2, 'color',[0.6 0.6 0.6]);
    end
    nicefigure;
    printlatex('../Report/graphics/Zoom_input_mobility_800Hz',6.5,10,'nofigcopy');
end

figure;
phase_H1 =angle(H1(2,:));
semilogx(f(20:end),phase_H1(20:end),'linewidth',1.2, 'color',[0.6 0.6 0.6]);
xlabel('Frequency (hertz)');
ylabel('Phase$\mathbf{Y}_{00}$ (radian)');
nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
xlim([20 f(end)]);
ylim([-pi/2*1.2 pi/2*1.2]);
set(gca,'Ytick',[-pi/2 -pi/4 0 pi/4 pi/2],'Yticklabel',{-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}});
nicefigure;
printlatex('../Report/graphics/Phase_input_mobility',13,10,'nofigcopy');

% Average transfer mobility of plates 1 to 4:
M_ms_v = zeros(1,size(Gvv,2)); % we define it previously in order to reduce computation load
for ii = 1:length(rho)-1
    M(ii) = rho(ii)*S(ii)*h(ii);
\begin{verbatim}
M_ms_v = M_ms_v + M(ii)*Gvv(ii,:);
third_f,third_M_ms_v = octave_third(f,M_ms_v);
end

Y_14 = M_ms_v./(Gff*sum(M));
figure;
semilogx(f,10*log10(Y_14),'LineWidth',1.2,'color',[0.6 0.6 0.6]);
nicefigure;
xlim([20 6400]);
nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
xlabel('Frequency (hertz)');
ylabel('$\left\langle |Y_t(f)|^2\right\rangle_{1-4}$ (\text{deci\hspace{1pt}bel})');

printlatex('../Report/graphics/Mobility_system1_plates_1_to_4',13,10,'nofigcopy');

% Input power
figure;
Ps = sum(Gff(20:end).*real_H1(2,20:end))*diff(f(1:2));
Ps_dB = 10*log10(Ps);%referred to 1W
semilogx(f(10:end),10*log10(Gff(10:end).*real_H1(2,10:end)),'LineWidth',1.2,'color',[0.6 0.6 0.6]);
nicefigure;
nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
xlim([100 6400]);
xlabel('Frequency (hertz)');
ylabel('$P(f)$ (\text{deci\hspace{1pt}bel \text{re} \unit{1}{watt}})');

printlatex('../Report/graphics/Power',13,10,'nofigcopy');

% vel_ratio == (h1/h2)^2 ==> same materials!!!
figure;
vel_ratio_f = Gvv(5,:)*sum(M)./M_ms_v;
semilogx([10 f(end)],[6 6],'LineWidth',1.2,'color',[0.3 0.3 0.3]);
hold on;
semilogx(f(10:end),10*log10(vel_ratio_f(10:end)),'LineWidth',1.2,'color',[0.6 0.6 0.6]);
legend('Theoretical','Measured');
nicefigure;
xlim([100 f(end)]);
nicelogx([64 125 250 500 1000 2000 4000 8000],'k');
xlabel('Frequency (hertz)');
ylabel('$\left\langle \overline{v_2(f)^2}\right\rangle\overline{\left\langle v_1(f)^2\right\rangle}$ (\text{deci\hspace{1pt}bel})');

printlatex('../Report/graphics/Theoretical_vs_measured_average_velocity_ratios',13,10,'nofigcopy');

third_ms_v_sys1 = third_M_ms_v/sum(M);
[third_f,third_Gvv] = octave_third(f,Gvv(5,:));
vel_ratio = third_Gvv./third_ms_v_sys1;
mean_vel_ratio = mean(vel_ratio(10:end)); % last page of the guide
figure;
semilogx(third_f,10*log10(vel_ratio), 'LineWidth',1.2,'color',[0.6 0.6 0.6]);
nicefigure;
nicelogx([16 32 64 125 250 500 1000 2000 4000 8000],'k');
%title('vel ratio (=4=6dB)');
hold on;
\end{verbatim}
151: semilogx([20 f(end)], [10*log10(4) 10*log10(4)], 'LineWidth', 1.2, 'color', [0.4 0.4 0.4]);
152: xlim([20 f(end)]);
153: xlabel('Frequency (hertz)');
154: ylabel('$\left\langle \overline{v_2(f)^2}\right\rangle/\left\langle \overline{v_1(f)^2}\right\rangle$ (decibel)');
155: legend('Measured', 'Theoretical', 'Location', 'SouthEast');
156: printlatex('../Report/graphics/mean_squared_velocities_ratio_third_oct', 13, 10, 'nofigcopy');
157:
158: % Damping factor:
159: ny = Gff.*real_H1(2,:)./(2*pi*f'.*(M_ms_v + S(5)*rho(5)*h(5)*Gvv(5,:)));
160: figure;
161: semilogx(f, ny, 'LineWidth', 1.2, 'color', [0.6 0.6 0.6]);
162: %title('damping factor');
163: ave_ny = mean(ny(100:6400));
164: nicefigure;
165: xlim([100 6400]);
166: nicefiglogx([64 125 250 500 1000 2000 4000 8000], 'k');
167: xlabel('Frequency (hertz)');
168: ylabel('$\eta(f)$');
169: title(sprintf('$\eta_{\text{average}}$=%1.2f', ave_ny));
170: printlatex('../Report/graphics/Measured_loss_factor', 13, 10, 'nofigcopy');
171:
172: save measurements real_H1 img_H1 Gvv Gff third_ms_v_sys1 vel_ratio Y_14 rho h S;

B.2. simul_theoretical_values.m

1: clear;
2: close all;
3: clc;
4:
5: poisson = 0.28;
6: E = 2e11;
7: f = 0:6.4e3; % Hz
8: S;
9: load measurements real_H1 img_H1 Gvv Gff third_ms_v_sys1 vel_ratio Y_14 rho h S;
10:
11: Y_14_measured = real_H1 + i*img_H1;
12: S_total_sys1 = sum(S(1:4));
13: M = rho(1)*h(1)*S_total_sys1;
14: ny = 0.01;
15: ny_5 = 0.003;
16:
17: % a)
18: B = E*h(1)^3/(12*(1-poisson^2)); %p. 117
19: real_Y = 1/(8*sqrt(B*rho(1)*h(1)));
20: figure;
21: semilogx([f(101) f(end)], [20*log10(real_Y) 20*log10(real_Y)], 'LineWidth', 1.0, 'color', [0.3 0.3 0.3]);
22: hold on;
23: semilogx(f(101:end), 20*log10(real_H1(2, 101:end))), 'LineWidth', 1.0, 'color', [0.6 0.6 0.6]);
24: legend('Theoretical', 'Measured', 'Orientation', 'horizontal', 'Location', 'NorthOutside');
25: legend boxoff
26: nicefigure;
27: xlim([101 6400]);
28: ylim([-85 -35]);
29: xlabel('Frequency (hertz)');
30: ylabel('Re$\{Y_{00}(f)\}^2$ (\deci\bel)');
31: printlatex('./Report/graphics/Theoretical_vs_measured_input_mobility',6.5,9,'nofigcopy');
32: 
33: \%
34: b)
35: Y_{14}\_theoretical = real\_Y/(2*pi*f*M*ny*(1+(ny_5*S(5)*h(1))/(ny*S_total\_sys1*h(5))));
36: figure;
37: semilogx([f(101) f(end)],[10*log10(Y_{14}\_theoretical(101)) 10*log10(Y_{14}\_theoretical(end))],'LineWidth',1.0,'color',[0.3 0.3 0.3]);
38: hold on;
39: semilogx(f(101:end),10*log10(abs(Y_{14}(101:end))),'LineWidth',1.0,'color',[0.6 0.6 0.6]);
40: legend('Theoretical','Measured','Orientation','horizontal','Location','NorthOutside')
41: legend boxoff
42: nicefigure;
43: xlim([101 6400]);
44: ylim([-85 -35]);
45: xlabel('Frequency (hertz)');
46: ylabel('$\left\langle\left|Y_t(f)\right|^2\right\rangle_{1-4}$ (\deci\bel)');
47: printlatex('./Report/graphics/Theoretical_vs_measured_transfer_mobility_Plates_1_to_4',6.5,9,'nofigcopy');
48: 
49: \%
50: c) average data
51: figure;
52: \[third\_f, ave\_real\_H1\] = octave_third(f,real\_H1);
53: semilogx([third\_f(10) third\_f(end)],[20*log10(real\_Y) 20*log10(real\_Y)],'LineWidth',1.0,'color',[0.3 0.3 0.3]);
54: hold on;
55: semilogx(third\_f(10:end),20*log10(ave\_real\_H1(10:end)),'LineWidth',1.0,'color',[0.6 0.6 0.6]);
56: legend('Theoretical','Measured','Orientation','horizontal','Location','NorthOutside')
57: legend boxoff
58: nicefigure;
59: xlim([third\_f(10) third\_f(end)]);
60: ylim([-85 -35]);
61: xlabel('Frequency (hertz)');
62: ylabel('Re$\{Y_{00}(f)\}^2$ (\deci\bel)');
63: printlatex('./Report/graphics/Theoretical_vs_measured_input_mobility\_third\_octaves',6.5,9,'nofigcopy');
64: 
65: \%
66: figure;
67: \[third\_f, ave\_Y_{14}\_measured\] = octave_third(f,Y_{14});
68: \[third\_f, ave\_Y_{14}\_theoretical\] = octave_third(f,Y_{14}\_theoretical);
69: semilogx([third\_f(10) third\_f(end)],[10*log10(ave\_Y_{14}\_theoretical(10)) 10*log10(ave\_Y_{14}\_theoretical(end))],'LineWidth',1.0,'color',[0.3 0.3 0.3]);
70: hold on;
71: semilogx(third\_f(10:end),10*log10(abs(ave\_Y_{14}\_measured(10:end)))),'LineWidth',1.0,'color',[0.6 0.6 0.6]);
Power, Vibration and Damping of a Box Structure

72: legend('Theoretical', 'Measured', 'Orientation', 'horizontal', 'Location', 'NorthOutside')
73: legend boxoff
74: nicefigure;
75: xlim([third_f(10) third_f(end)])
76: ylim([-85 -35])
77: nicegfigx([125 250 500 2000 4000 8000], 'k');
78: xlabel('Frequency (hertz)');
79: ylabel('$\langle |Y_t(f)|^2 \rangle_{1-4}$ (\decibel)');
80: printlatex('../Report/graphics/Theoretical_vs_measured_transfer_mobility_third_octaves', 6.5, 9, 'nofigcopy');
81:
82: [third_f, ave_Gvv] = octave_third(f, Gvv(5,:));
83: ave_vel_ratio = ave_Gvv./third_ms_v_sys1; %
84: % input power theory vs measurements
85: %
86: Ps_theory = sum(Gff(20:end).*real_Y)*diff(f(1:2));
87: Ps_theory_dB = 10*log10(Ps_theory); % referred to 1W
88: Ps_meas = sum(Gff(20:end).*real_H1(2,20:end))*diff(f(1:2));
89: Ps_meas_dB = 10*log10(Ps_meas); % referred to 1W

B.3. getNyquistData.m

1: function [Nf, NH] = getNyquistData(h, f, H)
2: %
3: disp('Select a mode by zooming. Press any key to continue');
4: pause;
5: %
6: % extract data in selected frequency range:
7: %
8: % frequency vector:
9: xlimits = get(h, 'XLim');
10: idx = find(f> xlim(1) & f<xlim(2));
11: Nf = f(idx);
12: %
13: % transfer function vector:
14: NH = H(idx);

B.4. nyquist_plot.m

1: function nyquist_plot(f, H, varargin);
2: %
3: if isempty(varargin)
4: plot_style = 'k-';
5: else plot_style = varargin{:};
6: end
7: %
8: figure;
9: plot(real(H), imag(H), plot_style);
10: axis([min(real(H))-0.1*max(real(H)) 1.1*max(real(H)) 1.2*min(imag(H)) 1.2*max(imag(H))]);
11: axis square;
12: axis equal;
B.5. NyquistFit.m

```matlab
function [f_res, f_l, f_u, damping] = NyquistFit(Nf, NH)
lw = 1.3;

idx = find(abs(NH) == max(abs(NH)));
maxNH = NH(idx);
f_res = Nf(idx);
hold on;
plot(real(maxNH), imag(maxNH), 'o', 'Color', [0.5 0.5 0.5]);
text(1.05*real(maxNH), imag(maxNH), '$f_0$');

% plot origin line to the point furthest away:
m = imag(maxNH)/real(maxNH); % slope
xvec = [0 real(maxNH)]; % new x-vector
OriginLine = m*xvec;
plot(xvec, OriginLine, '-', 'Color', [0.7 0.7 0.7], 'LineWidth', lw);

r_half = 1/2*sqrt(real(maxNH)^2 + imag(maxNH)^2);
l = sqrt(r_half^2 + r_half^2);

% first half of the circle:
alpha_1 = pi/4;
beta_1 = atan(imag(maxNH)/real(maxNH));
px_1 = l*cos(alpha_1 + beta_1);
py_1 = l*sin(alpha_1 + beta_1);
% dist_1 = sqrt(px_1^2 + py_1^2);
% plot(px_1, py_1, 'go');

f_1 = Nf(1:idx);
NH_1 = NH(1:idx);
p_cplx_1 = px_1 + i*py_1;
dist_cplx_1 = p_cplx_1 - NH_1;
idx_dNHmin_1 = find(abs(dist_cplx_1) == min(abs(dist_cplx_1)));
f_l = f_1(idx_dNHmin_1);
plot(real(NH_1(idx_dNHmin_1)), imag(NH_1(idx_dNHmin_1)), 'o', 'Color', [0.5 0.5 0.5]);
text(real(NH_1(idx_dNHmin_1)), 1.1*imag(NH_1(idx_dNHmin_1)), '$f_l$');

%second half of the circle:
px_2 = l*cos(beta_1 - alpha_1);
py_2 = l*sin(beta_1 - alpha_1);
% plot(px_2, py_2, 'mo');

f_2 = Nf(idx:end);
NH_2 = NH(idx:end);
p_cplx_2 = px_2 + i*py_2;
dist_cplx_2 = p_cplx_2 - NH_2;
idx_dNHmin_2 = find(abs(dist_cplx_2) == min(abs(dist_cplx_2)));
f_u = f_2(idx_dNHmin_2);
plot(real(NH_2(idx_dNHmin_2)), imag(NH_2(idx_dNHmin_2)), 'o', 'Color', [0.5 0.5 0.5]);
text(real(NH_2(idx_dNHmin_2)), 1.1*imag(NH_2(idx_dNHmin_2)), '$f_u$');
```

57: \% plot line perpendicular to the origin line:
58: \texttt{pm} = -1/m;
59: \texttt{offset} = \texttt{imag(maxNH)/2 - pm*real(maxNH)/2};
60: \% \texttt{newx} = \{px_1 px_2\};
61: \% \texttt{newy} = \{py_1 py_2\};
62: \texttt{newx} = \{\texttt{real(NH_1(idx_dNHmin_1)) real(NH_2(idx_dNHmin_2))}\};
63: \texttt{newy} = \{\texttt{imag(NH_1(idx_dNHmin_1)) imag(NH_2(idx_dNHmin_2))}\};
64:  
65: \texttt{pLine} = \texttt{pm*newx + offset};
66: \texttt{plot(newx, newy, '-', 'Color', [0.7 0.7 0.7], 'LineWidth', lw)};
67:  
68: \texttt{damping} = (f_u - f_l)/f_res;
69: \texttt{title(sprintf('$f_\text{0} = \text{\unit{\%4.4f}}{\hertz}$, $f_l = \text{\unit{\%4.4f}}{\hertz}$, $f_u = \text{\unit{\%4.4f}}{\hertz}$, $\eta = %1.4f$', f_res, f_l, f_u, damping));

References
